SYSE 596—Special Topic: Topological Methods in Systems Theory II

Syllabus
Summer Semester, 2026

Professor:	Dr. James Cale	Email:	jcale@colostate.edu
Office Hours:	By appointment	Phone:	(970) 412–0494

Course Description: This special topics course covers a survey of fundamental theorems in advanced mathematical topics from abstract algebra and topology, with application to systems theory and systems engineering. Part II of this course (2 credits hrs) covers various mathematical tools for describing objects, fields, and actions on discrete topological structures such as oriented graphs, simplicial complexes, and finite cell complexes. The course emphasizes algebraic representations of local quantities, boundary and coboundary operators, discrete Stokes-type principles, and local-to-global consistency of distributed data. The course includes individual reading, solving problems sets, literature reviews, and report assignments.

Prerequisites[1]

- Students must have taken previous coursework in senior-level undergraduate or graduate-level mathematics courses, with an emphasis on writing and understanding formal proofs.
  - This may also be satisfied by undergraduate or graduate degrees in physics or mathematics with coursework in spacetime geometry, and/or demonstration of the above background knowledge (to be evaluated by the instructor).

Reference Textbooks

Arfken, B. (2012). *Mathematical Methods for Physicists.* 7th ed. Academic Press.

Hartshorne, R. (1977). *Algebraic Geometry.* Springer-Verlag.

Hatcher, A. (2002). *Algebraic Topology.* Cambridge University Press.

Other Learning Materials

Instructor-provided course notes, recent research articles related to topological methods in systems theory.

Meeting Location and Time

This special topics course will not have a regular lecture component, but will consist of weekly meetings to assess students' progress. Regular meeting times will be established during the first session.

[1] Contact the instructor (jcale@colostate.edu) with questions and/or requests for waivers for the prerequisites.

Homework Assignments

Homework assignments will consist of problem sets from the course textbooks. These assignments will typically be due two weeks after distribution (dates/times will be listed on the assignment).

Report Assignments

This course includes two report assignments, each consisting of a short (5-10 page) written document and oral presentation. In these reports, students will be asked to explain the logical steps leading to one or more key theorems, written in standard mathematical format (definition, lemma, proof, definition, etc.). These reports may also include background material or theorems, requiring proper citation. Students must use LaTeX to produce their reports. Tentative dates for these reports are mid-semester (July 6th, 2026) and end-of-semester (Aug. 7th, 2026).

Course Grading Weights

Homework:	50%
Reports:	50%

Final Grade Assignments

Grade	**Score**
A+	96.67–100.00
A	93.33–96.66
A−	90.00–93.32
B+	86.67–89.99
B	83.33–86.66
B−	80.00–83.32
C+	76.67–79.99
C	70.00–76.66
D	60.00–69.99
F	0.00–59.99

AI Usage Policy

The use of artificial intelligence (AI) tools and technologies may be used for this course for research, to generate ideas, and as a tutor to help deepen your knowledge and understanding of the course content and learning objectives. You can use generative AI models (e.g., ChatGPT, Dall-E, Co-Pilot), neural networks, machine learning, and other techniques for these purposes, however, **you must cite and describe your AI use in your submitted work**. You may <u>not</u> submit material generated by AI that is misrepresented as your own, self-generated creative work. Refer to the APA guidelines for citing ChatGPT for citation standards[2]. The following are examples of permitted and prohibited usage.

[2] https://apastyle.apa.org/blog/how-to-cite-chatgpt

Permitted With Citation:

- Use of generative AI to provide an explanation or a synthesis of course content
- Use of generative AI to assist with grammar and readability of submitted work
- Use of generative AI to assist with concept ideation and research, where you are responsible for the formation and text of the submitted work
- Use of machine learning techniques for data analysis and to summarize results
- Use of generative AI to assist with coding or writing sections of code

Prohibited:

- Directly copying AI generated text (sentences, paragraphs, or other forms), code, or images and representing it as your own, i.e., without proper citation.

As the owner and author of your submitted work, you are fully responsible for all content, including that generated by or with AI tools. You are responsible for any inaccuracies, false information, inappropriate content, or any content that violates academic policies. Failure to cite or acknowledge AI use in your submitted work will constitute plagiarism. In cases where the use of AI tools is not fully cited and or questionable, you may be asked to provide evidence to demonstrate your knowledge and understanding of the submitted material.

Academic Integrity

The faculty expects every member of the CSU community to practice honorable and ethical behavior both inside and outside the classroom. Any actions that might unfairly improve a student's score on homework or examinations will be considered academic misconduct and will not be tolerated. Examples of academic misconduct include (but are not limited to):

- Requesting a regrade of answers or work that has been altered.
- Submitting work that is not your own.
- Representing as your own work anything that is the result of the work of someone or something else. This includes solutions obtained via solution manuals, the Internet, AI (without citation) and/or other services.

At the professor's discretion, academic misconduct on an assignment or examination/report will result in a reduced score, a zero score, or a failing grade for the course. All occurrences of academic misconduct will be reported to the Vice President for Student Affairs and copied to the Systems Engineering Department Head. If there is any question as to whether a given action might be construed as academic misconduct, please see the professor before you engage in any such action. For more information, please see CSU's page on Practicing Academic Integrity.[3] For information on the Honor Pledge, see the Honor Pledge.[4]

[3] http://learning.colostate.edu/integrity/
[4] http://tilt.colostate.edu/integrity/honorpledge/

SYSE 596—Special Topic: Topological Methods in Power Systems II

Learning Objectives

Students who successfully complete this course will be able to:

1. Use tensor notation, multilinear maps, dual spaces, bilinear forms, and contraction operations correctly in coordinate-based and coordinate-free descriptions of structured systems.
2. Define and use finite-dimensional vector spaces, linear maps, adjoints, kernels, images, quotients, and algebraic pairings in settings relevant to discrete field and network models.
3. Represent oriented graphs, simplicial complexes, and finite cell complexes algebraically through incidence relations, orientations, and boundary structure.
4. Construct chain and cochain spaces and compute the associated boundary and coboundary operators.
5. Explain and use the boundary and co-boundary operators and interpret their structural meaning in discrete models.
6. Relate discrete cochain constructions to the smooth viewpoint of differential forms, exterior differentiation, and Stokes' theorem.
7. Formulate and analyze discrete flow conservation, compatibility, and boundary-interaction laws using incidence structure, algebraic pairings, and discrete Stokes-type relations.
8. Develop a mathematically precise description of a system on a discrete topology using locally assigned algebraic data together with restriction-type and aggregation-type mechanisms for local-to-global consistency.

Topics by Week (Summer Offering)

Week	Topic
1	Review of tensor notation and multilinear algebra: index notation, contraction, tensor products, dual spaces, bilinear forms, and coordinate transformations
2	Algebraic structures for discrete modeling: vector spaces, linear maps, adjoints, quotients, and pairings; emphasis on quantities attached to vertices, edges, and higher cells
3	Oriented graph structures: incidence relations, directed graphs, paths, cycles, cuts, and conservation laws on networks
4	Graphs to cell complexes: simplicial complexes, cellular complexes, orientations, and discrete spaces as supports for algebraic data
5	Chains and boundary operators: chain spaces, boundary maps, and the identity $\partial^2 = 0$
6	Cochains and coboundary operators: dual spaces of chains, coboundary maps, and the identity $\delta^2 = 0$
7	Closed and exact structures: cycles, boundaries, cocycles, coboundaries, homological and cohomological interpretation on discrete topologies
8	Differential forms: motivation from smooth differential forms, exterior derivative, integration over oriented domains, and Stokes' theorem
9	Discrete Stokes-type principles: pairing of boundary and coboundary structure, interior cancellation, flux, circulation, and flow conservation
10	Local-to-global architectures (part I): locally assigned algebraic data, restriction maps, compatibility, and global consistency on discrete supports
11	Local-to-global architectures (part II): aggregation, extension, and cosheaf-style organization of distributed quantities; comparison with restriction-based architectures
12	Applications and synthesis: discrete models of networked and distributed physical systems, including boundary interaction, conservation structure, and local-to-global formulation, with in-depth example